

INDIAN SCHOOL MUSCAT
PRE-BOARD EXAMINATION
MATHEMATICS
CLASS : XII

	SET A		SET B		SET C	MARKS
1	(a) $x + 2y \leq 5; x + y \leq 4$	1	(b) 1	1	(b) 1	1
2	(d) 1	2	a) Feasible region	2	(b) $\frac{\pi}{2}$	1
3	(d) $\sqrt{5}$	3	(b) $\frac{x-1}{1} = \frac{y-2}{3} = \frac{z+3}{4}$	3	(d) Z	1
4	(d) Z	4	(d) $\frac{7\pi}{18}$	4	(a) $\pm \frac{2}{15}$	1
5	(a) $\frac{1}{2} + \frac{\pi}{4}$	5	(d) 126	5	(b) 1	1
6	(d) 3	6	(a) $x + 2y \leq 5; x + y \leq 4$	6	(d) -1	1
7	(b) 1	7	(a) 2	7	(b) 12π	1
8	(d) $\frac{7\pi}{18}$	8	(d) Z	8	(d) $\sqrt{5}$	1
9	(d) -1	9	(b) sec x	9	(a) 2	1
10	(a) Feasible region	10	(b) $\frac{1}{2}$	10	(d) 126	1
11	(b) $\frac{\pi}{2}$	11	(d) $\sqrt{5}$	11	(b) $\frac{1}{2}$	1
12	(b) sec x	12	(b) 1	12	(d) $\frac{7\pi}{18}$	1
13	(b) $\frac{1}{2}$	13	(a) $\pm \frac{2}{15}$	13	(a) $x + 2y \leq 5; x + y \leq 4$	1
14	(a) $\pm \frac{2}{15}$	14	(b) $\frac{\pi}{2}$	14	(b) sec x	1
15	(d) 126	15	(b) 12π	15	(a) $\frac{1}{2} + \frac{\pi}{4}$	1
16	(a) 2	16	(d) 1	16	(b) $\frac{x-1}{1} = \frac{y-2}{3} = \frac{z+3}{4}$	1
17	(b) 12π	17	(d) -1	17	(a) Feasible region	1
18	(b) $\frac{x-1}{1} = \frac{y-2}{3} = \frac{z+3}{4}$	18	(a) $\frac{1}{2} + \frac{\pi}{4}$	18	(d) 1	1
19	(c) A is true but R is false	19	(d) (A) is false but (R) is true	19	(a) both are true and R is correct explanation of A	1
20	(d) (A) is false but (R) is true	20	(d) (A) is false but (R) is true	20	(a) both are true and R is correct explanation of A	1

SET A	21.	$\vec{m} = \vec{a} + \vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ $\vec{n} = \vec{a} - \vec{b} = -\hat{j} - 2\hat{k}$ $\vec{m} \times \vec{n} = -2\hat{i} + 4\hat{j} - 2\hat{k}$ $ \vec{m} \times \vec{n} = 2\sqrt{6}$	{1/2 ½ ½}
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	$\vec{l} = \frac{9}{\sqrt{6}}(-\hat{i} + 2\hat{j} - \hat{k})$ OR Let $\hat{a} + \hat{b} = \hat{c}$ $\Rightarrow \hat{a} + \hat{b} ^2 = 1 + 1 + 2\hat{a} \cdot \hat{b}$ $\Rightarrow 2\hat{a} \cdot \hat{b} = -\frac{1}{2}$ $\Rightarrow \hat{a} - \hat{b} = \sqrt{3}$	1/2 1/2 1/2 1/2 1/2 1/2
22.	Put $x = \sin \theta \Rightarrow \theta = \sin^{-1} x$... (i) Let $y = \sin^{-1} [2x\sqrt{1-x^2}]$, $x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ $\Rightarrow y = \sin^{-1} [2\sin \theta \sqrt{1-\sin^2 \theta}]$ $\Rightarrow y = \sin^{-1} [2\sin \theta \sqrt{\cos^2 \theta}] = \sin^{-1} [2\sin \theta \cos \theta] = \sin^{-1} [2\sin \theta \cos \theta]$ $\Rightarrow y = \sin^{-1} [\sin 2\theta] = 2\theta$ $\therefore y = 2\sin^{-1} x$ [By (i)] Note that, $-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$ $\Rightarrow -\frac{1}{\sqrt{2}} \leq \sin \theta \leq \frac{1}{\sqrt{2}}$ $\Rightarrow -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$ and $-\frac{\pi}{2} \leq 2\theta \leq \frac{\pi}{2}$. OR $\pi/4 + 3\pi/4 + \pi/4 = 5\pi/4$	1/2 1/2 1/2 1/2 1/2 $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$
23.	$\frac{dy}{dx} = e^{x+y}$ $\Rightarrow e^{-y} dy = e^x dx$ $\Rightarrow \int e^{-y} dy = \int e^x dx$ $\Rightarrow -e^{-y} = e^x + C$	1 1/2 1/2
24.	Let $\frac{dV}{dt} = k$, where k is constant. As volume of the spherical ice-ball is $V = \frac{4}{3}\pi r^3$ $\Rightarrow \frac{dV}{dr} = 4\pi r^2$ $\therefore \left. \frac{dV}{dr} \right _{at\ r=10\ cm} = 4\pi \times 10^2 = 400\pi \text{ cm}^3/\text{cm}$.	1/2 1/2 1/2 1/2
25.	$y = a e^{2x} + b e^{-x}$ $\frac{dy}{dx} = 2a e^{2x} - b e^{-x}$ $\frac{d^2y}{dx^2} = 4a e^{2x} + b e^{-x}$ $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$	1/2 1/2 1/2 1/2 1/2
26.	$\frac{(x^2 + 1)}{(x-1)^2(x+3)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+3)}$	1

	A = 3/8, B = 1/2 C = 5/8 $\frac{3}{8} \log x-1 - \frac{1}{2(x-1)} + \frac{5}{8} \log x+3 + C$	1 1
27.	$\int \frac{x+3}{(x+5)^3} e^x dx$ $= \int \frac{(x+5)-2}{(x+5)^3} e^x dx$ $= \int \left(\frac{x+5}{(x+5)^3} - \frac{2}{(x+5)^3} \right) e^x dx$ $= \int \left(\frac{1}{(x+5)^2} - \frac{2}{(x+5)^3} \right) e^x dx$ $= \frac{e^x}{(x+5)^2} + C$	1 1/2 1 1/2
28.	<p>The feasible region determined by the constraints, $x+2y \geq 100$, $2x-y \leq 0$, $2x+y \leq 200$, $x, y \geq 0$, is given below.</p> <p>$A(0, 50)$, $B(20, 40)$, $C(50, 100)$ and $D(0, 200)$ are the corner points of the feasible region.</p>	1 1/2 for 3 lines 1/2 for shading feasible region

The values of Z at these corner points are given below.

1/2

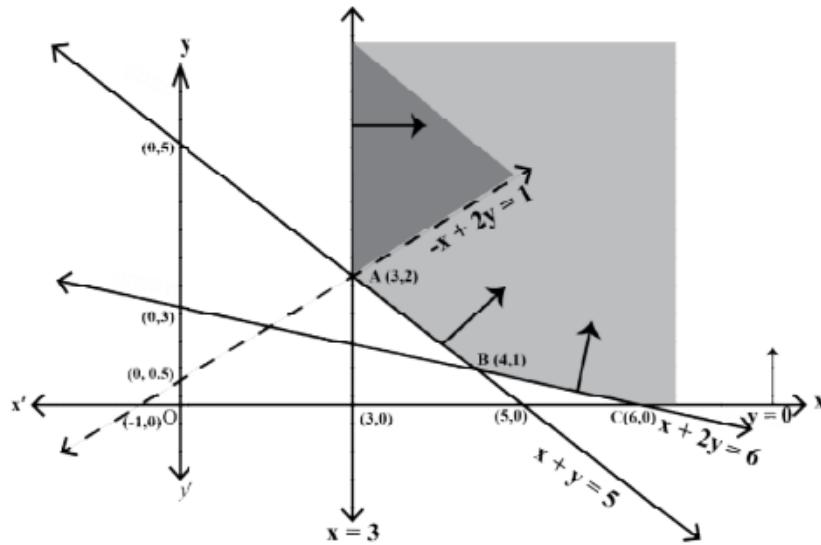
Corner point	Corresponding value of $Z = x + 2y$	
A (0, 50)	100	Minimum
B (20, 40)	100	Minimum
C (50, 100)	250	
D (0, 200)	400	

1/2

The minimum value of Z is 100 at all the points on the line segment joining the points (0,50) and (20,40).

OR

The feasible region determined by the constraints, $x \geq 3, x + y \geq 5, x + 2y \geq 6, y \geq 0$. is given below.



1 1/2

Here, it can be seen that the feasible region is unbounded.

The values of Z at corner points A (3, 2), B (4, 1) and C (6, 0) are given below.

1/2

Corner point	Corresponding value of $Z = -x + 2y$
A (3, 2)	1 (may or may not be the maximum value)
B (4, 1)	-2
C (6, 0)	-6

1/2

Since the feasible region is unbounded, $Z = 1$ may or may not be the maximum value.

Now, we draw the graph of the inequality, $-x + 2y > 1$, and we check whether the resulting open half-plane has any point/s, in common with the feasible region or not.

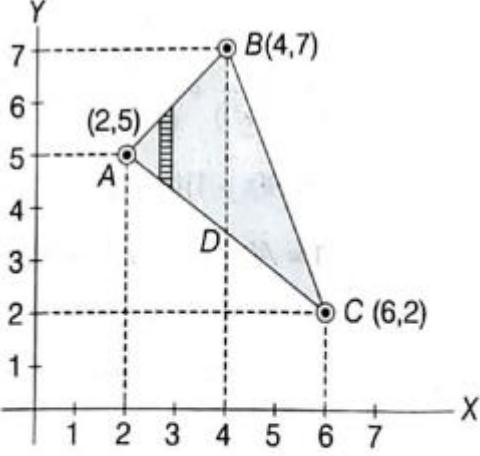
1/2 for dotted line

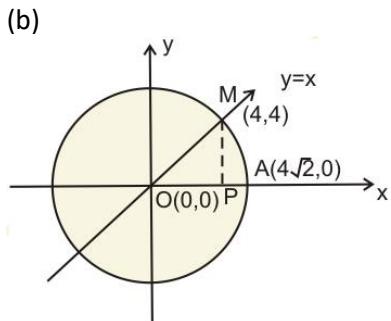
Here, the resulting open half plane has points in common with the feasible region.

1/2 for conclusion

Hence, $Z = 1$ is not the maximum value. We conclude, Z has no maximum value.

<p>29.</p> $\text{Let } I = \int_0^{\pi/2} \frac{dx}{1 + \sqrt{\tan x}}$ $\Rightarrow I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \dots(\text{i})$ $I = \int_0^{\pi/2} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{2} - x\right)} + \sqrt{\sin\left(\frac{\pi}{2} - x\right)}} dx$ $= \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \dots(\text{ii})$ <p>Adding (i) and (ii), we get</p> $2I = \int_0^{\pi/2} 1 \cdot dx$ $\Rightarrow I = \frac{\pi}{4}$	½ ½								
<p>30.</p> <p>Let X be no. of selected scouts who are well trained in first aid. Here random variable X may have value 0, 1, 2.</p> <p>Now, $P(X = 0) = \frac{\binom{20}{2}}{\binom{50}{2}} = \frac{20 \times 19}{50 \times 49} = \frac{38}{245}$</p> <p>$P(X = 1) = \frac{\binom{20}{1} \times \binom{30}{1}}{\binom{50}{2}} = \frac{20 \times 30 \times 2}{50 \times 49} = \frac{120}{245}$</p> <p>$P(X = 2) = \frac{\binom{30}{2}}{\binom{50}{2}} = \frac{30 \times 29}{50 \times 49} = \frac{87}{245}$</p> <p>Now probability distribution table is</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 5px;">X</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">2</td> </tr> <tr> <td style="padding: 5px;">$P(x)$</td> <td style="padding: 5px;">$\frac{38}{245}$</td> <td style="padding: 5px;">$\frac{120}{245}$</td> <td style="padding: 5px;">$\frac{87}{245}$</td> </tr> </table> <p style="text-align: center;">OR</p> <p>Let A be the event that a student reads Hindi newspaper and B be the event that a student reads English newspaper.</p> <p>$P(A) = 60/100 = 0.6$, $P(B) = 40/100 = 0.4$ and $P(A \cap B) = 20/100 = 0.2$</p> <p>(a) Now $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $= 0.6 + 0.4 - 0.2$ $= 0.8$</p> <p>Probability that she reads neither Hindi nor English newspaper $= 1 - P(A \cup B)$ $= 1 - 0.8$ $= 0.2$ $= 1/5$</p>	X	0	1	2	$P(x)$	$\frac{38}{245}$	$\frac{120}{245}$	$\frac{87}{245}$	½ ½ 1 ½ ½
X	0	1	2						
$P(x)$	$\frac{38}{245}$	$\frac{120}{245}$	$\frac{87}{245}$						

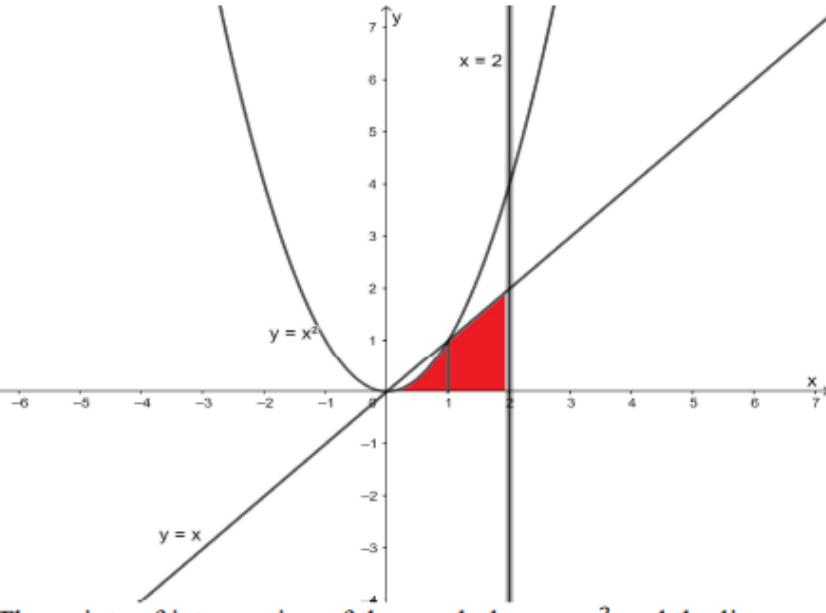
	34.	<p>(i) Volume of open box = $x(24 - 2x)^2$ (ii) Local max at c_1 (iii) $v = x(24 - 2x)^2 \frac{dV}{dx} = 0 \Rightarrow x = 4$ Therefore, side of square is 4cm. Or $v = x(24 - 2x)^2$, put $\frac{dv}{dx} = (24 - 2x)(24 - 6x)$ $= 0, x = 4$ $\left(\frac{d^2y}{dx^2}\right)_{x=4} = -6(24 - 2x) - 2(24 - 6x) = -96$ $< 0, \text{max}$ $V_{\text{MAX}} = 4(24-8)^2 = 1024 \text{ Sq units}$</p>	1 1 1 + 1 1 1 1
	35.	 <p>Equations of AB, BC and AC</p> <p>Integration</p> <p>Area = 7 sq. units</p> <p>OR</p>	1 1 1½ 2 ½



	$\text{Area} = \int_0^4 x \, dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2} \, dx$ $= \left(\frac{x^2}{2} \right)_0^4 + \left(\frac{x\sqrt{32 - x^2}}{2} + \frac{32}{2} \sin^{-1}\left(\frac{x}{4\sqrt{2}}\right) \right)_4^{4\sqrt{2}}$ $= 4\pi$	1 1 1
36.	$AB = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$ $x - y = 3,$ $2x + 3y + 4z = 17,$ $y + 2z = 7$ $\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ $\Rightarrow A X = C$ $\Rightarrow X = A^{-1} C$ <p>Since $AB = 6I$</p> $\Rightarrow A^{-1} = B/6$ $= \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ $\text{So } X = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ <p>Thus $x = 2, y = -1, z = 4$</p>	1 1 ½ ½ ½ 1 ½
	OR	
	$ A = -1 \neq 0 \text{ so } A^{-1} \text{ exist}$ <p>For finding correct cofactors</p> $\text{Correct } A^{-1} = \begin{bmatrix} 0 & -2 & -1 \\ 1 & 9 & 5 \\ -2 & -23 & -13 \end{bmatrix}$ $X = A^{-1}B, \quad x = 1, y = 2, z = 3$	1 1 ½ 1 ½ 1
37.	reflexive symmetric Transitive Equivalence	1.5 1.5 1.5 ½

	38.	<p>The given line is $\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$ Its cartesian eq. is $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1} = \lambda \text{ (say)} \quad \dots(i)$ Any point Q on (i) is $(2\lambda - 1, 3\lambda + 3, -\lambda + 1)$ Also, the given point is $P(5, 4, 2)$. Now d.r's of the line PQ are $(2\lambda - 1 - 5, 3\lambda + 3 - 4, -\lambda + 1 - 2) = (2\lambda - 6, 3\lambda - 1, -\lambda - 1)$. For PQ to be \perp to (i), we must have $(2\lambda - 6) \cdot 2 + (3\lambda - 1) \cdot 3 + (-\lambda - 1) \cdot (-1) = 0$ $\Rightarrow 14\lambda - 14 = 0 \Rightarrow \lambda = 1$ $\therefore Q$ is $(1, 6, 0)$ which is the foot of \perp from P on line (i). $\begin{aligned} \text{Now, } PQ &= \sqrt{(5-1)^2 + (4-6)^2 + (2-0)^2} \\ &= \sqrt{24} = 2\sqrt{6} \text{ units.} \end{aligned}$ Further if $R(\alpha, \beta, \gamma)$ is the image of P in line (i), then $\begin{aligned} \frac{\alpha+5}{2} &= 1, \frac{\beta+4}{2} = 6, \frac{\gamma+2}{2} = 0 \\ \Rightarrow \alpha &= -3, \beta = 8, \gamma = -2 \\ \therefore \text{Image of } P \text{ in line (i)} &= R(-3, 8, -2). \end{aligned}$ </p>	$\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$
B	23	<p>(a)</p> <p>Given $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ $\Rightarrow \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c} = 0$ $\Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0$ \Rightarrow either $\vec{b} = \vec{c}$ or $\vec{a} \perp \vec{b} - \vec{c}$</p> <p>Also given $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ $\Rightarrow \vec{a} \times \vec{b} - \vec{a} \times \vec{c} = 0 \Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = 0$ $\Rightarrow \vec{a} \parallel \vec{b} - \vec{c}$ or $\vec{b} = \vec{c}$</p> <p>But vector a cannot be both parallel and perpendicular to (vector $b-c$). Hence vector $b=c$.</p>	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

B	25	$f(x) = 4x^3 - 6x^2 - 72x + 30$ implies $f'(x) = 12x^2 - 12x - 72$ $12x^2 - 12x - 72 < 0$ $12(x^2 - x - 6) < 0$ $x^2 - x - 6 < 0$ $x^2 - 3x + 2x - 6 < 0$ $x(x - 3) + 2(x - 3) < 0$ $(x - 3)(x + 2) < 0$ $x \in (-2, 3)$	$\frac{1}{2}$
	26	$\int_0^4 x - 1 dx = \int_0^1 (1 - x) dx + \int_1^4 (x - 1) dx$ $= [x - \frac{x^2}{2}]_0^1 + [\frac{x^2}{2} - x]_1^4$ $= (1 - \frac{1}{2}) + (8 - 4) - (\frac{1}{2} - 1)$ $= 5$	1 1 1
B	29 (a)	$ydx + (x - y^2)dy = 0$ we get, $\frac{dx}{dy} + \frac{x}{y} = y$ I.F = $e^{\int P dy} = e^{\int \frac{1}{y} dy} = e^{\log y} = y$ $x \cdot IF = \int Q \cdot IF dy \Rightarrow xy = \int y^2 dy$ $\Rightarrow xy = \frac{y^3}{3} + C$, which is the required general solution	$\frac{1}{2}$ 1 1 $\frac{1}{2}$
B	36.	Let $(a, b) \in N \times N$. Then we have $ab = ba$ (by commutative property of multiplication of natural numbers) $\Rightarrow (a, b)R(a, b)$ Hence, R is reflexive. Let $(a, b), (c, d) \in N \times N$ such that $(a, b) R (c, d)$. Then $ad = bc$ $\Rightarrow cb = da$ (by commutative property of multiplication of natural numbers) $\Rightarrow (c, d)R(a, b)$ Hence, R is symmetric.	1 $\frac{1}{2}$

	<p>Let $(a, b), (c, d), (e, f) \in N \times N$ such that $(a, b) R (c, d)$ and $(c, d) R (e, f)$. Then $ad = bc$, $cf = de$ $\Rightarrow adcf = bcde$ $\Rightarrow af = be$ $\Rightarrow (a, b)R(e, f)$ Hence, R is transitive. Since, R is reflexive, symmetric and transitive, R is an equivalence relation on $N \times N$.</p>	$\frac{1}{2}$ $\frac{1}{2}$
37(b)	 <p>The points of intersection of the parabola $y = x^2$ and the line $y = x$ are $(0, 0)$ and $(1, 1)$.</p> <p>Required Area = $\int_0^1 y_{\text{parabola}} dx + \int_1^2 y_{\text{line}} dx$</p> <p>Required Area = $\int_0^1 x^2 dx + \int_1^2 x dx$</p> $= \left[\frac{x^3}{3} \right]_0^1 + \left[\frac{x^2}{2} \right]_1^2 = \frac{1}{3} + \frac{3}{2} = \frac{11}{6}$	(Correct Fig: 1 Mark) $\frac{1}{2}$ 2 $\frac{1}{2}$

C	30 (b)	$x dy - y dx = \sqrt{x^2 + y^2} dx$ It is a Homogeneous Equation as $\frac{dy}{dx} = \frac{\sqrt{x^2 + y^2} + y}{x} = \sqrt{1 + (\frac{y}{x})^2} + \frac{y}{x} = f\left(\frac{y}{x}\right).$ Put $y = vx$ $\frac{dy}{dx} = v + x \frac{dv}{dx}$ $v + x \frac{dv}{dx} = \sqrt{1 + v^2} + v$ Separating variables, we get $\frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$ Integrating, we get $\log v + \sqrt{1 + v^2} = \log x + \log K, K > 0$ $\log y + \sqrt{x^2 + y^2} = \log x^2 K$ $\Rightarrow y + \sqrt{x^2 + y^2} = \pm K x^2$ $\Rightarrow y + \sqrt{x^2 + y^2} = C x^2$, which is the required general solution	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$1+1/2$
C	31.	$\int \frac{dx}{\sqrt{3 - 2x - x^2}}$ $= \int \frac{dx}{\sqrt{-(x^2 + 2x - 3)}} = \int \frac{dx}{\sqrt{4 - (x+1)^2}}$ $= \sin^{-1}\left(\frac{x+1}{2}\right) + C \quad [\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C]$	2	1		